Spintessence! New Models for Dark Matter and Dark Energy

Latham A. Boyle[1], Robert R. Caldwell[2], and Marc Kamionkowski[1]
[1]*Mail Code 130-33, California Institute of Technology, Pasadena, CA 91125*
[2]*Department of Physics & Astronomy, Dartmouth College, Hanover, NH 03755*
(May 2001)

We investigate a class of models for dark matter and/or negative-pressure, dynamical dark energy consisting of "spintessence," a complex scalar field ϕ spinning in a $U(1)$-symmetric potential $V(\phi) = V(|\phi|)$. As the Universe expands, the field spirals slowly toward the origin. The internal angular momentum plays an important role in the cosmic evolution and fluctuation dynamics. We outline the constraints on a cosmic spintessence field, describing the properties of the potential necessary to sustain a viable dark energy model, making connections with quintessence and self-interacting and fuzzy cold dark matter. Possible implications for the coincidence problem, baryogenesis, and cosmological birefringence, and generalizations of spintessence to models with higher global symmetry and models in which the symmetry is not exact are also discussed.

PACS numbers: 98.80.Cq, 95.35.+d, 98.65.Dx, 98.70.Vc

Introduction. Supernova evidence [1] for an accelerating Universe has been dramatically bolstered by the discrepancy between the total cosmological density $\Omega_{\rm tot} \simeq 1$ indicated by the cosmic microwave background (CMB) [2] and dynamical measurements of the nonrelativistic-matter density $\Omega_m \simeq 0.3$. New and independent evidence is provided by higher peaks in the CMB power spectrum that also suggest $\Omega_m \simeq 0.3$ [2], again leaving 70% of the density of the Universe unaccounted for. As momentous as these results are for cosmology, they may be even more remarkable from the vantage point of fundamental physics since they indicate the existence of some form of negative-pressure "dark energy".

For this dark energy to accelerate the expansion, its equation-of-state parameter $w \equiv p/\rho$ must satisfy $w < -1/3$, where p and ρ are the dark-energy pressure and energy density, respectively. The simplest guess for this dark energy is the spatially uniform, time-independent cosmological constant for which $w = -1$. Another possibility is quintessence [3], a cosmic scalar field [4] that is displaced from the minimum of its potential. Negative pressure is achieved when the kinetic energy of the rolling field is less than the potential energy, so that $-1 \le w < -1/3$ is possible.

This negative-pressure dark energy should not be confused with the cold dark matter that has long been known to be required to support flat galactic rotation curves and to provide the majority of the matter in galaxy clusters. Leading candidates for this dark matter include collisionless particles such as supersymmetric particles [5] and the axion [6]. However, numerical simulations of structure formation with collisionless dark matter seem to indicate more galactic substructure than is observed [7], a discrepancy that has led some to postulate that the dark matter might possess a self-interaction [8] or consist of extremely low-mass particles ("fuzzy" dark matter) [9].

In this paper, we consider a new class of models for dark matter and dark energy. We investigate the behavior of a complex scalar field that is spinning in a circular orbit in a $U(1)$-symmetric potential $V(\phi) = V(|\phi|)$, a monotonically increasing function of $|\phi|$. As the Universe expands, the radius of this orbit, and thus the potential- and kinetic-energy densities decrease. It is the internal-angular-momentum barrier, not expansion friction, that prevents the field from falling directly to the minimum of the potential. Unlike quintessence models, spintessence allows $|\phi|$ to change slowly even if the time derivative of ϕ is large. As well, the growth of perturbations in spintessence differs from those in quintessence or cold dark matter.

Below, we discuss the evolution of spintessence and the growth of perturbations, working through some simple illustrative examples. We conclude with some remarks about the viability of spintessence models with global symmetries other than $U(1)$ or in the presence of broken global symmetry, and we mention possible links to quintessence, baryogenesis, and other areas of particle physics and early-Universe cosmology.

Spintessence. We can decompose a complex scalar field into two real fields: $\phi(\mathbf{x},t) = \phi_1(\mathbf{x},t) + i\phi_2(\mathbf{x},t) \equiv R(\mathbf{x},t)\exp[i\Theta(\mathbf{x},t)]$. First suppose that ϕ is homogeneous, lives in Minkowski space, and has a $U(1)$-symmetric potential-energy density $V = V(|\phi|)$ that is a monotonically increasing function of $|\phi|$. Then its equations of motion are equivalent to those of a classical particle moving in a two-dimensional central potential $V(R)$. The simplest non-trivial solutions are those in which the field moves in a circular orbit, $\phi(t) = Re^{i\omega t}$, with R and ω constants that satisfy $R\omega^2 = V'(R)$ so the centripetal acceleration balances the radial force.

In an expanding Universe, conservation of the global-charge current means $\dot\Theta = Q/a^3R^2$ where Q is a constant associated with the total charge, and $a(t)$ is the cosmological scale factor. With regards to the field dynamics, the charge introduces a secular driving-term into the equation-of-motion for R,

$$\ddot{R} + 3H\dot{R} + V'(R) = \frac{Q^2}{a^6 R^3} \quad (1)$$

where $H = \dot{a}/a$. If the spin frequency is high, $\dot{\Theta} \gg H$, we may expect the rotation to dominate, supporting the field against radial infall. In this rapidly-spinning approximation, the time evolution of R is then determined from $V'(R) = Q^2 a^{-6} R^{-3}$. From this we find that the potential must satisfy $(d/dR)[R^3 V'(R)] > 0$ if it is to be steep enough to confine the field to a circular orbit as the Universe expands. For instance, with a quadratic potential, $R \propto a^{-3/2}$ in a matter-dominated epoch so that the radial kinetic energy rapidly decays $\dot{R}^2 \propto a^{-6}$, leaving energy density and pressure

$$\rho = \frac{1}{2}(\dot{R}^2 + R^2\dot{\Theta}^2) + V, \quad p = \frac{1}{2}(\dot{R}^2 + R^2\dot{\Theta}^2) - V, \quad (2)$$

with $R^2\dot{\Theta}^2 = 2V \propto a^{-3}$, and an equation-of-state $w = 0$. For such rapidly spinning fields, the equation-of-state parameter is

$$w(R) \approx \frac{RV'(R) - 2V(R)}{RV'(R) + 2V(R)}. \quad (3)$$

However, solutions with an arbitrary constant equation-of-state, for which each term in ρ, p above decays as $\propto a^{-3(1+w)}$, are not possible owing to the conserved charge.

Growth of perturbations. We now consider the growth of perturbations in spintessence. While the perturbations in a spinning field have been considered (for different purposes) in Refs. [10,11] for the special case of quadratic and quartic potentials, here we generalize their analysis to arbitrary potentials. We start with the spacetime line element

$$ds^2 = (1+2\Phi)dt^2 - (1-2\Phi)a^2(t)d\mathbf{x}^2. \quad (4)$$

where $\Phi(\mathbf{x}, t)$ is the Newtonian potential arising from fluctuations in the spinning field, $R(t) + \delta R(\mathbf{x}, t)$ and $\Theta(t) + \delta\Theta(\mathbf{x}, t)$, and surrounding matter. The evolution equations for the perturbations, δR and $\delta\Theta$, obtained from the linearized Einstein Equations are

$$\begin{aligned}
\delta\ddot{R} + 3\frac{\dot{a}}{a}\delta\dot{R} + &\left(V'' - \dot{\Theta}^2 - \frac{1}{a^2}\nabla^2\right)\delta R \\
&= 4\dot{R}\dot{\Phi} - 2\Phi V' + 2R\dot{\Theta}\delta\dot{\Theta}, \quad (5)\\
\delta\ddot{\Theta} + 3\frac{\dot{a}}{a}\delta\dot{\Theta} - &\frac{1}{a^2}\nabla^2\delta\Theta \\
&= 4\dot{\Theta}\dot{\Phi} - 2\frac{\delta\dot{R}}{R}\dot{\Theta} + 2\frac{\dot{R}}{R}\left(\frac{\delta R}{R}\dot{\Theta} - \delta\dot{\Theta}\right), \quad (6)\\
\nabla^2\Phi - 3H\dot{\Phi} - &3H^2\Phi \\
&= 4\pi G\big[\dot{R}\delta\dot{R} + V'\delta R + R^2\dot{\Theta}\delta\dot{\Theta} \\
&\quad + R\dot{\Theta}^2\delta R - \Phi\big(\dot{R}^2 + R^2\dot{\Theta}^2\big)\big]. \quad (7)
\end{aligned}$$

The final line gives the constraint equation to the gravitational potential. The stability of a real scalar field depends on the effective mass, V''. But here we see that the stability criteria for the spinning field must differ since not only is the effective mass different, $V'' - \dot{\Theta}^2$, but also the δR and $\delta\Theta$ equations are coupled. Before proceeding to a full-blown relativistic calculation, we can infer essential information about the behavior of perturbations for the rapidly spinning field with a Newtonian analysis set in Minkowski spacetime. There, perturbations to the gravitational potential of the form

$$\Phi(\mathbf{x}, t) = \Phi_1 e^{\Omega t + i\vec{k}\cdot\vec{x}} \quad (8)$$

will be generated through the Poisson equation (7) by small amplitude perturbations to the amplitude and phase of the scalar field,

$$\delta R(\mathbf{x}, t) = R_1 e^{\Omega t + i\vec{k}\cdot\vec{x}}, \quad \delta\Theta(\mathbf{x}, t) = \Theta_1 e^{\Omega t + i\vec{k}\cdot\vec{x}}. \quad (9)$$

Leaving out terms that are small for $k^2 \gtrsim G\rho$, corresponding to wavelengths inside the horizon, we obtain the wavenumber k_J at which $\Omega^2 = 0$. For $k < k_J$, where k_J is the Jeans wavenumber

$$k_J^2 = \frac{1}{2}\left[\frac{V'}{R} - V'' + \sqrt{\left(\frac{V'}{R} - V''\right)^2 + 64\pi G V'^2}\right] \quad (10)$$

then $\Omega^2 > 0$, and the perturbations grow exponentially. For $k > k_J$ then $\Omega^2 < 0$, and the perturbations oscillate in time. Note that these instabilities will be effective in an expanding Universe as long as we consider the following: (1) The physical wavelength associated with a given comoving wavelength changes with time. (2) Perturbations on scales larger than the horizon will be stabilized. (3) The time dependence of unstable perturbations inside the horizon will be power law rather than exponential.

If the spintessence field is to supply a dark matter component, then the existence of a gravitational instability, whether exponential or power-law, is welcome. As a dark energy candidate, however, we require stability against the growth of perturbations on scales at least as large as clusters. As a complex field with a conserved charge, Q, the spintessence field is susceptible to the formation of Q-balls — a nontopological soliton [12]. In order to avoid the formation of Q-balls from the spinning field, however, it is necessary that the field does not have an instability that drives $R \to R_{qb}$, the non-zero value of the field amplitude at which the quantity $V(R)/R^2$ has a minimum. Checking with (3), we note when $w = 0$, and provided $V''(R_{qb}) > 0$, the conditions are ripe for the formation of Q-balls. This means that a rapidly spinning field cannot safely pass through a $w = 0$ phase. This consideration places a further constraint on the behavior of a viable spintessence model of dark energy. Note that if the spinning field does not entirely decay into Q-balls, there is the interesting possibility that both dark energy and dark matter might consist of the spinning field, which would provide an interesting possibility for solving the coincidence problem.

We now consider specific examples of spintessence potentials.

Power-Law: A rapidly spinning field in a potential $V(R) = V_0(R/R_0)^n$ has a constant equation-of-state parameter $w = (n-2)/(n+2)$. The quadratic $U(1)$ potential leads to a matter density that decays as cold dark matter, as the dynamics of ϕ_1 and ϕ_2 are those for two decoupled harmonic oscillators. Since $V'/R - V'' = 0$ we find $k_J^2 \simeq 4\sqrt{\pi G}V'$, and the instability is driven by gravity. Perturbations on smaller scales are stabilized by scalar-field dynamics. Such a field is unstable to the formation of Q-balls, which may provide for an interesting dark matter component. On the other hand, potentials with $n > 2$ are stable against Q-ball formation. The Jeans scale is $k_J^2 \simeq 16\pi G R V'/(n-2)$, provided $GR^2 \ll 1$. However, $w > 0$ so that these are less interesting from the dark matter / dark energy perspective.

A dark energy component with a $n < 1$ power-law potential is plagued by an instability which leads to the formation of Q-balls. Since a dominant component necessarily has $V \sim (m_{pl}H)^2$, we find the Jeans wavenumber is $k_J \sim H m_{pl}/R$. Although R might start out with an amplitude comparable to the Planck mass, since R necessarily decays as the Universe expands, it is inevitable that the wavenumber will eventually be well within the Hubble horizon. The behavior $dk_J/dR < 0$ as R decays disqualifies a wide class of potentials as dark energy.

Self-interacting and fuzzy cold dark matter. Suppose $V(R) = \frac{1}{2}m^2R^2 + \frac{\lambda}{4}R^4$. If $\lambda > 0$, then $w = 1/3$ at early times when $\lambda R^4/4 \gg m^2R^2/2$, but approaches $w = 0$ at later times. For $\lambda < 0$ we consider only values of the scalar field $R < m/\sqrt{-\lambda}$, and in this case, there is a negative pressure that approaches $w = 0$ at late times as the quartic term becomes small. If the quartic term is small, then this describes a gas of cold massive particles that self-interact via a repulsive ($\lambda > 0$) or attractive ($\lambda < 0$) potential. After collapse and virialization of halos, either type of interaction would give rise to a plausible self-interacting dark-matter candidate. The homogeneous and perturbation analysis above can be used to determine how the mean density and perturbations to this type of dark matter would evolve with time.

Now consider the fuzzy cold dark matter of Ref. [9]. They suppose that halo dark matter consists of a quadratic potential of mass m. They adopt a value $m \sim 10^{-22}$ eV to smooth galactic halos and since this dark matter must contribute a density $\rho \sim (10^{-3}\,\mathrm{eV})^4 \sim m^2R^2$, they must have $R \sim 10^{16}\,$eV. This gives rise to a wavenumber $k_J \sim 10^{-28}$ eV. More generally, however, there should be a nonzero quartic term in the potential, but if the dark matter is to be cold, the quartic term must be small compared with the quadratic term. This leads to a constraint $|\lambda| \lesssim 10^{-76}$. As small as this is, the condition of validity $[(V'/R) - V'']^2 \ll 64\pi G V'^2$ for their estimate of the Jeans scale is even more restrictive; it leads to $|\lambda| \lesssim 10^{-87}$. Thus, if $10^{-87} \lesssim |\lambda| \lesssim 10^{-76}$, then the Jeans wavenumber is $10^{-28} \lesssim k_J \lesssim 10^{-22}$ eV for $\lambda < 0$, or $10^{-34} \lesssim k_J \lesssim 10^{-28}$ eV for $\lambda > 0$. Thus, the inclusion of a small nonzero quartic interactions can spread the Jeans length over 11 orders of magnitude.

Dark Energy: A spintessence field must meet a number of constraints in order to be considered as a viable dark energy candidate. All together, these conditions may be summarized as: $0 < RV' < V$ to ensure the existence of circular orbits (lower bound) and equation of state $w < -1/3$ (upper bound); $-\frac{1}{3}R^2V'' < RV' < R^2V''$ requiring steep orbits (lower) and stable perturbations in the absence of gravity (upper). In the presence of gravity, of course, we use $k_J \lesssim H$ with equation (10) in order to assess stability. Recall that $V(R)$ need not satisfy these conditions for all R, just in the range $R_{max} > R > R_{min}$, the values at which field evolution begins at early times, and the value today. Potentials which satisfy the above criteria include $V(R) = M^2R^2(A + (R/B)^{-r})\mathrm{e}^{-1/R^2}$ with $1 < r < 2$, or $V(R) = (M^2R^2 - A)\mathrm{e}^{-BR^2} + A$ both with $A, B > 0$, as suggested by Kasuya [13]. Along the same lines, a potential of the form $V = M^4\exp[m^2/(R_{max}^2 - R^2)]$ can give rise to a stable, dark energy component. In the regime $R < min(R_{max}, R_{qb})$ where $R_{qb} = -m/2 + \sqrt{R_{max}^2 + m^2/4}$, the field can evolve for a long time with $w < 0$, before it is necessary to patch on a different functional form for V at some small value of the field amplitude, say R_{min}, to ensure $V(0) = 0$. A stability analysis reveals that the quantity $V'/R - V''$ is negative, unlike the power-law potential, which immediately tells us from equation (10) that gravity will play the dominant role in determining the Jeans wavenumber. Plugging in our potential, we find $k_J \sim M^2\sqrt{R}/(m_{pl}R_{max})$. Since a dominant component has $V \sim (m_{pl}H)^2$, then the Jeans wavenumber reduces to $k_J \sim H\sqrt{m_{pl}R}/R_{max}$ which is substantially outside the Hubble horizon for $R \ll R_{max}$. The perturbations are stable. Lastly, there is a novel twist to a dark energy scenario based on the above potentials. When the field passes to $R < R_{min}$, the accelerated expansion ends. If the potential in this regime possesses a minimum in V/R^2, then the dark energy field will ultimately decay into Q-balls.

Discussion. We have considered a class of models for dark energy and dark matter that consists of a complex scalar field spinning in a $U(1)$ potential. Specification of $V(R)$ determines the scaling of the equation-of-state and density as a function of redshift, and it also determines how density perturbations grow. These solutions are valid if the spin frequency is $\dot{\Theta} \gg H$. If the spin period is small, $\dot{\Theta} \lesssim H$, then the field will act like quintessence and will undergo friction-dominated slow rolling toward the minimum of the potential. Thus, depending on the potential, a model may begin as quintessence and wind up like spintessence, or *vice versa.* Fluctuations

in spintessence will differ from a real scalar field, nevertheless, due to the greater number of excitable degrees of freedom. Spintessence could conceivably be used to drive inflation [11], although it is difficult to see how the large global charge density, or alternatively high spin frequency, could be maintained during the many e-folds of expansion required for inflation. Perhaps a greater difficulty is how to set up a homogeneous, spinning field at the end of inflation.

If the dark energy is due to spintessence, then the Universe is in an unstable state that breaks T and C invariance. This suggests interesting connections between the dark-energy problem and other questions in cosmology and particle physics. For example, if the global charge of the spintessence field is identified with baryon number, then spintessence may be the vacuum that hides the antibaryons in a baryon-symmetric baryogenesis model [14]. Alternatively, spintessence could conceivably drive baryogenesis in an Affleck-Dine or spontaneous-baryogenesis model [15]. If the field is coupled to the pseudoscalar of electromagnetism, it could give rise to P- and T-violating rotations of polarization of cosmological sources [16].

Spintessence should work for higher global symmetries [e.g., $O(N)$ with $N > 2$], as orbits are still confined to a surface in the internal space in such models. Although heuristic arguments suggest that quantum gravity should violate global symmetries at least to some degree [17], the basic idea of spintessence should still work. As a simple example, suppose that $V(\phi_1, \phi_2) = c_1\phi_1^n + c_2\phi_2^n$ with $c_1 \neq c_2$. Although orbits in this potential are not circular and there is no conserved internal angular momentum, the virial theorem guarantees that when averaged over an orbit, the potential-energy density T and kinetic-energy density V will still be related by $T = (n/2)V$. Thus, as long as the dynamical time for the potential is small compared with the expansion time, the equation-of-state should still behave like that for spintessence.

In summary, spintessence is the simplest example of a cosmological field with a non-trivial internal symmetry group. We have outlined the constraints which must be satisfied to obtain a viable cosmological model, making connections with quintessence and varieties of fuzzy- and self-interacting dark matter. We have shown that adding the internal symmetry gives rise to a rich collection of new phenomena: different clustering properties, an instability to Q-balls, and a new way to drive the acceleration, via the angular momentum barrier as opposed to Hubble friction. If Q-balls or similar objects are an inevitable by-product of a cosmological field with a non-trivial internal symmetry group, then it would seem that the viability of spintessence relies on the compatibility of Q-balls with cosmology.

LB was supported by an NSF Graduate Fellowship. This work was supported at Caltech by NSF AST-0096023, NASA NAG5-8506, and DoE DE-FG03-92-ER40701 and DE-FG03-88-ER40397. The work at Dartmouth was supported by NSF-PHY-0099543.

Note: During the preparation of this paper, several other papers [13,18–21] appeared that also consider a spinning complex scalar field with respect to dark matter and dark energy.

---

[1] S. Perlmutter et al., Astrophys. J. **517**, 565 (1999); A. G. Riess et al., Astron. J. **116**, 1009 (1998).
[2] M. Kamionkowski, D. N. Spergel, and N. Sugiyama, Astrophys. J. Lett. **426**, L57 (1994); A. D. Miller et al., Astrophys. J. **524**, L1 (1999); P. de Bernardis et al., Nature **404**, 955 (2000); S. Hanany et al., Astrophys. J. Lett., in press [astro-ph/0005123]; A. H. Jaffe et al., astro-ph/0007333; C. B. Netterfield et al., astro-ph/0104460; C. Pryke et al., astro-ph/0104490.
[3] R. R. Caldwell, R. Dave, and P. J. Steinhardt, Phys. Rev. Lett. **80**, 1582 (1998).
[4] B. Ratra and P. J. E. Peebles, Phys. Rev. D **37**, 3406 (1988); C. Wetterich, Astron. & Astrophys. **301**, 321 (1995); K. Coble, S. Dodelson, and J. Frieman, Phys. Rev. D **55**, 1851 (1997); M. S. Turner and M. White, Phys. Rev. D **56**, 4439 (1997).
[5] G. Jungman, M. Kamionkowski, and K. Griest, Phys. Rep. **267**, 195 (1996).
[6] See, e.g., M.S. Turner, Phys. Rep. **197**, 67 (1990); G. G. Raffelt, Phys. Rep. **198**, 1 (1990); L. J. Rosenberg and K. A. van Bibber, Phys. Rep. **325**, 1 (2000).
[7] B. Moore et al., Astrophys. J. Lett. **524**, L19 (1999); A. A. Klypin, A. V. Kravtsov, and O. Valenzuela, astro-ph/9901240.
[8] D. N. Spergel and P. J. Steinhardt, Phys. Rev. Lett. **84**, 3760 (2000).
[9] W. Hu, R. Barkana, and A. Gruzinov, Phys. Rev. Lett. **85**, 1158 (2000).
[10] M. Yu. Khlopov, B. A. Malomed, and Ya. B. Zeldovich, Mon. Not. R. Astron. Soc. **215**, 575 (1985).
[11] P. Jetzer and D. Scialom, Phys. Rev. D **55**, 7440 (1997).
[12] S. Coleman, Nuc. Phys. B. **262**, 263 (1985).
[13] S. Kasuya, Phys. Lett. B **515**, 121 (2001).
[14] S. Dodelson and L. Widrow, Phys. Rev. Lett. **64**, 340 (1990); Phys. Rev. D **42**, 326 (1990).
[15] I. Affleck and M. Dine, Nucl. Phys. B **249**, 361 (1985); A. G. Cohen and D. B. Kaplan, Phys. Lett. B **199**, 251 (1987).
[16] S. Carroll, Phys. Rev. Lett. **81**, 3067 (1998); A. Lue, L. Wang, and M. Kamionkowski, Phys. Rev. Lett. **83**, 1506 (1999).
[17] M. Kamionkowski and J. March-Russell, Phys. Rev. Lett. **69**, 1485 (1992); Phys. Lett. B **282**, 137 (1992); R. Holman et al., Phys. Rev. Lett. **69**, 1489 (1992); Phys. Lett. B **282**, 132 (1992).
[18] J.-An Gu and W.-Y. P. Hwang, astro-ph/0105099.
[19] A. Arbey, J. Lesgourgues and P. Salati, Phys. Rev. D **64**, 123528 (2001).
[20] X. Z. Li, J. G. Hao and D. J. Liu, astro-ph/0107171.
[21] T. H. Chiueh, Phys. Rev. D **65**, 123502 (2002).